REPORT

on the procedure for the defense of a dissertation entitled: "Functionals of Lévy Processes and Their Applications" for obtaining the educational and scientific degree "Doctor" by Martin Yordanov Minchev

Field of higher education: 4. Natural Sciences, Mathematics, and Informatics.

Professional field: 4.5. Mathematics.

Doctoral program: "Probability Theory and Mathematical Statistics", Department: Probability, Operational Research, and Statistics (PORS), Faculty of Mathematics and Informatics (FMI), Sofia University "St. Kliment Ohridski" (SU).

The report is prepared by Prof. D.Sc. Ljuben Radoslavov Mutafchiev, emeritus professor at the American University in Bulgaria, in the capacity of a member of the scientific jury, according to Order No. RD-38-200/26.04.2024 of the Rector of SU.

1. General Characteristics of the Dissertation Thesis and Other Submitted Materials

The present dissertation (in English) contains 175 pages and is divided into 6 chapters and 2 supplementary sections (appendices). In the first chapter, entitled "Introduction," are provided some necessary facts from the theory of Lévy processes and increment processes, and in the last chapter, entitled "Conclusion," the candidate lists the new results that he has obtained in his thesis and provides information related to some formal requirements (publication, approval of results, declaration of authenticity). The bibliography contains 172 items.

At the beginning of the dissertation, the author expresses his gratitude to individuals and a foundation for their support. In addition to the dissertation thesis and its abstract (in Bulgarian and English), the other submitted materials include 23 documents (diplomas for bachelor's and master's degrees, a copy of an article related to the dissertation and published in the prestigious international journal Bernoulli, a report from the scientific advisors, the candidate's CV, a summary of compliance with the minimum national requirements for obtaining the educational and scientific degree "Doctor", orders for enrollment and discharge from the doctoral program, a protocol from a meeting of the PORS department to initiate the procedure for public defense, an internal review of the dissertation, and others).

2. Data and Personal Impressions of the Candidate

Martin Minchev was born in January, 1994, in Shumen. He graduated from the Sofia Mathematical High School in 2013. During 2014-2015, he has participated in an undergraduate program on mathematics and physics at the Lycée Louis-le-Grand in Paris (France), and in 2018, he obtained a bachelor's degree on applied mathematics from FMI of SU. Subsequently, during 2018-2019, Martin Minchev studied at Sorbonne University, Paris, where he obtained his master's degree on probability and stochastic models. He successfully defended his thesis under the supervision of Prof. Justin Salez. Since October 2020, Martin Minchev has been enrolled in a full-time doctoral program at FMI. From 2021 to the present, he has participated in several scientific projects. For short periods, he has interned at the Bulgarian National Bank (2017) and some international firms (2017, 2020, and 2021), where he worked as a researcher and quantitative analyst. He has also been an R1 level researcher under the "Young Scientists" program (2021) and an adjunct assistant at FMI of SU.

I am well acquainted with the topic on which Martin Minchev is working. I have participated in the scientific jury for the academic position "professor" of his scientific advisors. I have also attended several seminar presentations and lectures related to the topic and the results obtained in the present thesis.

3. Analysis of the Scientific Achievements of the Candidate Which Are Included in the Present Dissertation and in the Associated Publication

It is well known that Lévy processes present a natural generalization of random walk processes with continuous time parameter. An exponential functional of a Lévy process is an integral of the negative exponential function whose argument is the underlying Lévy process, where the integration is over the time parameter with lower bound zero and upper bound a deterministic or random moment, which can be also equal to infinity. This integral is a random variable that contains information about the overall evolution of the given process. The study of the properties of this variable is a major problem in the theory of Lévy processes. In general, the study of the exponential functionals was initiated by the Polish mathematician K. Urbanik. It turns out that these functionals are a major tool and play an important role of both a theoretical and a practical in the study of self-similar random processes, Asian options in financial mathematics, random processes in random environment, branching processes with immigration, and others. In the material of the present dissertation the candidate studies exponential functionals of Lévy processes which is an important and current area from the theory of random processes. The dissertation is written correctly and competently. The main part of its material is contained in Chapters II-V. Below I will outline some personal impressions in more detail.

1. I believe that the candidate has put a lot of effort and work to create a good format of the texts of the dissertation (in English) and the abstracts (in Bulgarian and English). In the dissertation, he used a style which is typical for mathematical publications.
2. The dissertation contains a detailed review in the field of functionals of Lévy processes, where the candidate clearly distinguishes his contributions from the results obtained by other authors. In the review material, he clearly explains why the results obtained in the dissertation are important for the theory of Lévy processes.
3. The large amount of journal literature that the candidate has studied and reviewed is impressive.
4. The candidate has provided complete proofs of the new results obtained in his dissertation. He has used there both probabilistic methods and facts from the theory of Lévy processes, as well as asymptotic and analytical methods. Significant technical difficulties have been overcome in applying the saddle point method. Classical and modern Tauberian theorems and Mellin transformations in the complex plane have also been used.

   I am fully convinced that the candidate Martin Minchev possesses the qualities of a good mathematician with solid background of knowledge and skills.

Next, I will focus on the main scientific contributions in the dissertation:

- An exponential functional of a Lévy process with an upper bound that is an exponentially distributed random variable, independent of the Lévy process, is considered. Under relatively weak conditions on the process, asymptotic expressions for the density and its derivatives of this exponential functional, as well as for its tail distribution, have been obtained.
- A Lévy process with a finite negative mean value and a tail distribution that is asymptotically equivalent to a given slowly varying function divided by a power function with an exponent greater than 1 is considered. Under these conditions, weak convergence of the appropriately scaled exponential functional of the process has been obtained, where the scaling significantly depends on the given slowly varying function. As a consequence, the

asymptotics of some functional transformations of the exponential functional of the given process have been obtained.

- The applications of the Mellin transform in the study of the properties of exponential functionals of Lévy processes requires to introduce the so-called Bernstein-gamma function. There is an analogy between this function and the classical gamma function. As an analogue of the classical Stirling formula for the gamma function, the candidate has obtained an exact estimate for the Bernstein-gamma function in the complex plane under the assumption that the real part of its argument becomes large.
- Bivariate Bernstein-gamma functions which extend the class of the aforementioned univariate ones above have also been considered. New integral representations of these functions in terms of the harmonic measure of the associated Lévy process have been found, and necessary and sufficient conditions for the finiteness of their derivatives have been obtained.

I believe that the dissertation contains new and important results, which present a solid contribution to the modern theory of random processes.

4. Approval of the Results

Martin Minchev has presented one publication which is essentially related to the material of his dissertation. It was published in 2023 in the prestigious journal Bernoulli, published by the Bernoulli Society for Mathematical Statistics and Probability. The article is co-authored with Mladen Savov, who has declared that both co-authors have equal contribution to its preparation. I am pleased to note that this article has already been cited in a doctoral dissertation from Cornell University (USA) and in 6 articles published in reputable specialized journals and series: Stochastic Processes Appl., Ann. H. Lebesgue, Ann. Inst. Henri Poincare Probab. Statist., Lecture Notes Math., Proc. London Math. Soc., Electron. Commun. Probab. At the end of his dissertation, Martin Minchev has attached a declaration on the originality of the results obtained by him or in collaboration with his co-author, stating that the use of results from other scientists is always accompanied by the appropriate citation. The candidate has reported parts of his dissertation at 3 scientific events in our country and has participated twice with a poster presentation at conferences in Portugal and the United Kingdom.

I think that the published article meets the minimum national requirements (under Art. 2b, Para. 2 and 3 of the Law on the Development of the Academic Staff in the Republic of Bulgaria), as well as the additional requirements of SU for obtaining the educational and scientific degree "Doctor" in the scientific field and professional direction of the procedure. No proven plagiarism has been established in the present dissertation and in the scientific article related to this procedure.

5. Qualities of the Abstract

The abstract correctly, accurately, and thoroughly presents the results and the content of the dissertation. The enclosed bibliography is complete and confirms the high competence of the candidate.

6. Critical Remarks and Recommendation

I have no essential critical remarks regarding the dissertation and the accompanying article. As far as I know, the "saddle point method" is translated into Bulgarian as "метод на седловата точка" rather than "метод на седлото."

7. Conclusion

After reviewing the present dissertation and the accompanying scientific work and based on the analysis of their significance and the scientific and applied scientific contributions they contain, I confirm that the present dissertation, the accompanying scientific publication, as well as the quality and originality of the results and achievements presented in them, meet the requirements of the Law on the Development of the Academic Staff in the Republic of Bulgaria (ZRASRB), its Regulations for application, and the relevant Regulations of the Sofia University "St. Kliment Ohridski" for awarding the educational and scientific degree "Doctor" in the scientific field 4. Natural Sciences, Mathematics, and Informatics, and the professional field 4.5. Mathematics. In particular, the candidate satisfies the minimum national requirements in the professional field, and no plagiarism has been found in the scientific works he has submitted for the procedure.

Based on the above, I recommend to the scientific jury to award Martin Yordanov Minchev the educational and scientific degree "Doctor" in the scientific field 4. Natural Sciences, Mathematics, and Informatics, professional field 4.5. Mathematics (Probability Theory and Mathematical Statistics).

Prepared by: Prof. D.Sc. Lyuben Mutafchiev

10.06.2024